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A METHOD OF CHOOSING PROJECTILE MANUFACTURING TOLERANCES SO AS TO MINIMIZE COSTS OF PRODUCTION WHILE SATISFYING FUNCTIONAL REQUIREMENTS

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DECEMBER 1970

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A method has been devised which allows one to compute the tolerances of dimensions of a projectile or other manufactured item in such a manner as to minimize the costs of production. The method further allows the imposition of any number of inequality constraints (tolerances) on properties of the dimensions (weight, volume, center of mass position, etc.), which will be satisfied to a linear approximation.

A particular form (hyperbolic) for the cost function has been chosen as the example here computed, but the method is not limited to this form. A computer program to facilitate numerical application of this technique has been written, and another program to compute the required sensitivity coefficients for three dimensional mass asymmetry limits is presently in development.

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SUMMARY

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INTRODUCTION

Certain problems of dimensional irregularity encountered during the production of the 175mm, M437 Projectile were brought to the attention of this laboratory. The effects of these irregularities on the static and dynamic unbalances and the effect of these unbalances on the flight of these projectiles were not sufficiently well understood to justify acceptance or rejection of the projectiles in question. During the course of the investigation of the flight dynamic effects of these irregularities (which did not fall within manufacturing tolerances) it became clear that an opportunity existed for the introduction of more sophisticated engineering methods into the decision making process. These methods ought to be useful for evaluation of requests for waiver of tolerance.

The following analysis is an early result of a search for superior techniques for choosing projectile tolerances.

DISCUSSION

If the properties of an item one wishes to control in manufacture are called Z^i and these Z^i are functions of the dimensions of the item, X^i, and the minimum and maximum acceptable values of Z^i are C_1^i, and C_2^i, one may say

$$C_1^i \leq Z^i[\, X^k,\ k=1,n] \leq C_2^i \qquad (1)$$

If we assume that any variation in any actual dimension of an item is small compared to the nominal dimension itself, then

$$\Delta Z^i \approx \sum_{k=1}^{n} \frac{\partial Z^i}{\partial X^k} \Delta X^k \qquad (2)$$

where the $\frac{\partial Z^i}{\partial X^k}$ are evaluated at the nominal dimensions of the item, and are called sensitivity coefficients.

Combining Equations 1 and 2 we obtain, approximately

$$\delta C_1^i \leq \sum_{k=1}^{n} \frac{\partial Z^i}{\partial X^k} \Delta X^k \leq \delta C_2^i \qquad (3)$$

where $\delta C_1^i = C_1^i - Z^i_{nominal} \leq 0$, $\delta C_2^i = C_2^i - Z^i_{nominal} \geq 0$

and $\Delta Z^i = Z^i - Z^i_{nominal}$, $\Delta X^k = X^k - X^k_{nominal}$

In order to assure that

$$\sum_{k=1}^{n} \frac{\partial Z^i}{\partial X^k} \Delta X^k \leq \delta C_2^i$$

One may require that

$$\frac{\partial z^i}{\partial x^k} \Delta x^k \leq f_k \, \delta C_2^i \tag{4}$$

where the f_k are not a priori known but obey

$$0 \leq f_k \leq 1 \tag{5}$$

$$\sum_{k=1}^{n} f_k = 1 \tag{6}$$

and, similarly, to assure that

$$\sum_{k=1}^{n} \frac{\partial z^i}{\partial x^k} \Delta x^k \geq \delta C_1^i$$

we require that

$$\frac{\partial z^i}{\partial x^k} \Delta x^k \geq f_k \, \delta C_1^i$$

with the f_k obeying Equations 5 and 6.

Assuming that the f_k be somehow determined for each control property, z^i, the minimum and maximum excursion of the Δx^k can be written

$$\Delta x^k_{i_{min}} \equiv \frac{\delta C_1^i f_k}{\frac{\partial z^i}{\partial x^k}} \qquad \frac{\partial z^i}{\partial x^k} > 0. \tag{8}$$

$$\equiv \frac{\delta C_2^i f_k}{\frac{\partial z^i}{\partial x^k}} \qquad \frac{\partial z^i}{\partial x^k} < 0.$$

and

$$\Delta x^k_{i_{max}} \equiv \frac{\delta C_2^i f_k}{\frac{\partial z^i}{\partial x^k}} \qquad \frac{\partial z^i}{\partial x^k} > 0. \tag{9}$$

$$\equiv \frac{\delta C_l^i f_k}{\frac{\partial Z^i}{\partial X^k}} \qquad \frac{\partial Z^i}{\partial X^k} < 0.$$

from Equations 4 and 7.

If these be interpreted as limits on the range of X^k due to the limits imposed on Z^i, the intersection over all i of the regions

$$\Delta X^k_{i_{min}} \le \Delta X^k \le \Delta X^k_{i_{max}}$$

will have bounds δX^k_{min} and δX^k_{max} and still obey all requirements within the approximation (Eq 2). These are the intersections of all the sets of restrictions:

$$\delta X^k_{min} = \max_i \, (\Delta X^k_{i_{min}}) \tag{10}$$

$$\delta X^k_{max} = \min_i \, (\Delta X^k_{i_{max}}) \tag{11}$$

The absolute minimum and maximum may be chosen at this point although the f_k are not known. Since for a given k all the $\Delta X^k_{i_{max}}$ contributing to δX^k_{max} <u>have the same</u> f_k (and different C's and $\frac{\partial Z^i}{\partial X^k}$'s), the minimum $\Delta X^k_{i_{max}}$ is the one with minimum $\frac{\delta C}{\frac{\partial Z^i}{\partial X^k}}$. The same argument holds for δX^k_{min}.

So for a given projectile and specifications [Z, X, C], δX^k_{min} and δX^k_{max} are constants times f_k.

It is now necessary to introduce the notion of a cost function, which reflects the unit cost of production with a given process or sequence of processes, and depends on the tolerances required. Given the equipment and size of the lot, the fixed costs are determined. Call this ξ_0 .

It is apparent that there exists some tolerance which is not quite possible to obtain using the ℓth process; call it η_ℓ. The production cost grows very large as the tolerance.

on this process, $\delta X^{\ell}_{max} - \delta X^{\ell}_{min}$, approaches this η_{ℓ} and the cost function must reflect this tendency. Further, each process is different and each has its own constant, ξ_{ℓ}, multiplying its contribution to the total cost. Therefore we define a cost function \$.

$$\$ = \xi_o + \Sigma \frac{\xi_{\ell}}{\delta X^{\ell}_{max} - \delta X^{\ell}_{min} - \eta_{\ell}} \qquad (12)$$

where the ξ_{ℓ} are provided; they reflect the cost of reducing the tolerance on a given process; they need not be given in absolute terms, but relative to each other: i.e., $\xi_j = 2.3\ \xi_{j-1}$; and the η_{ℓ} are the lowest practical value of the tolerance of the ℓth operation. So the problem proposed is to minimize \$ by a suitable choice of the f_k subject to the conditions $\Sigma f_j = 1$, $f_j \geq 0$. Note that the δX^j are linear in f_j and are functions only of f_j and the constants of the minimization problem. C^i_1, C^i_2, $\frac{\partial Z^i}{\partial X_j}$ are all fixed for a given projectile. Therefore we propose to minimize

$$\$ = \xi_o + \sum_{\ell=1}^{n} \frac{\xi_{\ell}}{k_{\ell} f_{\ell} - \eta_{\ell}} \qquad (13)$$

where η_{ℓ} is the tightest tolerance possible by the ℓth manufacturing operation, and ξ_{ℓ} is the rate of change of cost with respect to tolerance level for the ℓth operation, subject to the condition

$$\sum_{\ell=1}^{n} f_{\ell} = 1 \Leftrightarrow \sum_{\ell=1}^{n} (f_{\ell} - \frac{1}{n}) = 0 \qquad (6a)$$

Equations 5,6a, and 13 can be combined into

$$\$ = \xi_o + \Sigma \left[\frac{\xi_{\ell}}{k_{\ell} f_{\ell} - \eta_{\ell}} + \Pi \ (f_{\ell} - \frac{1}{n})\right] \qquad (14)$$

where Π is a Lagrange multiplier ($\neq 0$) and the usual condition for an extremum with respect to f_j is applied:

$$\frac{\partial \$}{\partial f_j} = \frac{-\xi_j k_j}{[k_j f_j - \eta_j]^2} + \Pi = 0,$$

which is n equations in the n f's and one Π and

$$\sum_{\ell=1}^{n} f_\ell = 1 \qquad \text{is the } n+1^{th} \text{ equation.}$$

Solving for f_j and Π yields:

$$f_j = \frac{\sqrt{\frac{\xi_j k_j}{\Pi} + \eta_j}}{k_j} = \sqrt{\frac{\xi_j}{k_j \Pi} + \frac{\eta_j}{k_j}} \qquad (15)$$

and

$$\Pi = \left\{ \frac{\sum_{j=1}^{n} \sqrt{\xi_j/k_j}}{1 - \sum_{j=1}^{n} \eta_j/k_j} \right\}^2 \qquad (16)$$

Therefore, the actual tolerances can be calculated from these closed form solutions by substituting actual values into Equations 8, 9, 15, and 16, and the results into Equations 10 and 11.

CONCLUSIONS

It has been demonstrated that it is possible to optimize the cost of production of a projectile in a manner which guarantees approximate satisfaction of any number of inequality constraints on any property of the dimensions (the functional requirements).

This is, of course, in some sense a "worst case" solution since an item with maximum error in all its dimensions still satisfies all the inequality constraints. This may lead to impractically tight tolerances. It is then clear that some small but finite failure rate (fail-

ure to meet the functional requirements) is acceptable in a lot, if all of the items in it satisfy the tolerances imposed on the dimensions.

We believe that the above computed distribution of tolerance (that is, the relative sizes of the tolerance) is still useful. Further, it is felt that a suitable constant for each tolerance can be developed which will involve the probability of each half-distribution of dimension exceeding tolerance, normalized in such a way that each half distribution will contribute an equal amount toward the probability of not satisfying the functional requirements.

Further effort in this direction is necessary for a more complete understanding of the probabilistic effects and this effort will be pursued.